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DEPARTMENT OF THE ARMY
US ARMY RESEARCH, DEVELOPMENT AND ENGINEERING COMMAND
ARMY RESEARCH LABORATORY
2800 POWDER MILL ROAD
ADELPHI MD 20783-1197

REPLY TO
ATTENTION OF

April 5, 2017

Office of Chief Counsel

SUBJECT: Freedom of Information Act (FOIA) Request (FP-17-007297/FA-17-0003) for document entitled: Research on Technical Applications of Extreme Values; Accession Number: AD0280119; Author(s): Gumbel, Emil J., Columbia University New York School of Engineering And Applied Science; Report Date: 01 Apr 1962; Descriptive Note: Final Report

Mr. John Greenwald
[redacted]
john@greenwald.com

Dear Mr. Greenwald:

This is in response to your Freedom of Information Act request, dated September 29, 2016, which was received by the undersigned on January 4, 2017 for the subject document.

Attached are the documents responsive to your request in their entirety, a total of 31 pages, fully releasable without redactions.

Fees in this matter have been waived in accordance with DOD Directive 5400.7-R.

If you have any questions, you may contact Edith Koleoso at the above address, by telephone at (301) 394-1636, or by e-mail to ARL-FOIA-Request@arl.army.mil.

Sincerely,

Timothy W. Connolly
FOIA Officer
U.S. Army Research Laboratory

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Final Report
April 1 1962

RESEARCH ON

TECHNICAL APPLICATIONS OF EXTREME VALUES

Contract No. DA 30 069 Ord 1061

ASTIA
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Statistical Engineering Group
Department of Industrial and Management Engineering
School of Engineering
Columbia University in the City of New York

FOREIGN ANNOUNCEMENT AND DISSEMINATIO
OF THIS REPORT BY ASTIA IS LIMITED.

Technical Supervisor
Office of Ordnance Research

Project Director, Professor Sebastian B. Littauer
Principal Investigator, Professor Emil J. Gumbel

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Research on Contract DA 30-069 ORD 1061 with Columbia University, which began on 1 February 1953, was initiated in response to requests from General Leslie E. Simon and Charles Bicking of the then Office of Ordnance Research to provide methods of analysis of strength of materials. Their aim in encouraging this research was to improve the quality of materials of military materiel and, in particular, the materiel in production under Army Ordnance. The expectation that the methods of extreme values would provide methods beyond those of the statistics of the Normal, Binomial, Poisson and other more familiar distributions for analysing the behavior of materials under various forms of stress has been realized in twenty-two technical reports given in Appendix B. Six of these reports result from the first two years work on this contract from 1 February 1953 to 30 June 1955, which forms an integral part of these researches. Many publications have also appeared and they are given in Appendix C.

The primary applications of the statistical theory of extreme values have been to breaking strength and fatigue of materials. Other applications have been made to hydrology In order, however, to provide the techniques which were effective in these applications it was necessary to conduct research in general statistical theory as well as in the statistical theory of extreme values. These researches, then, have four main categories:

I. General Statistical Theory

II. Statistical Theory of Extreme Values

III. Applications to Breaking Strength and Fatigue of Materials

IV. Applications to Hydrology.

A roster of technical personnel attached to this contract is given in Appendix A. Other scientific work engaged in by workers on this contract is given in Appendix D. Finally acknowledgement is made of the services of Mrs. Mary Roston who assisted the Principal Scientist and Project Director in carrying out many of the necessary administrative duties.

I. GENERAL STATISTICAL THEORY

Some of the phenomena of extreme value theory have significance in general statistical theory. One such useful concept is the "Return Period of Order Statistics", (Technical Report 21, publication 43). The return period T(X) is a mean number of observations on X such that for some value x, the expected number among such observations which are equal to or greater than x is equal to one. If the observations all occur at equidistant times the return period has the dimension of time. Hence the name return period. This is expressed formally as follows: if F(X) is the cumulative distribution function of X and T(X) is the return period for $X \geq x$, then

$$(1) \quad T(x) = \left(1 - F(x)\right)^{-1}$$

Although the return period is defined as an expected value verbally, its determination is as a conditional variable. Nevertheless, the return period can be studied under assumptions that the chance variable is both distribution and parameter free and, therefore, the return period of an order statistic is of general statistical interest.

Suppose we consider an order statistic of rank m, where the maximum observation in a sample of size n has rank 1. Then one obtains for the expected value of k^{th} moment T_m^k of the return period T_m of the order statistic of rank m, the expression

$$(2) \qquad E(T_m^k) = \frac{n(n-1)\ \ldots\ (n-k+1)}{(m-1)\ (m-2)\ \ldots\ (m-k)} \quad .$$

This expression is finite only for $k \leq m-1$. An interesting particular result obtains for n = 2, where the expected return period is two for the second order statistic but is infinite for the first order statistic. Furthermore, the variance is infinite for either statistic.

Since "rare" events are formally representable as events of low probability they lend themselves to extreme value analysis. This connection has been shown in Technical

Report 14 which was published in a work on measurement theory (publication 29). An interesting consequence of the statistical analysis of rare events is that attempts have been made by researchers in extrasensory perception to validate their findings thereby. Richard von Mises probed this, so to speak, quasi scientific field with his armor of scientific insight and statistical analysis. E. J. Gumbel prepared an article from von Mises' findings which is to be published in his collected works.

Several studies have been devoted to bivariate distributions. As a remnant of nineteenth century opinions many statisticians believe that the so-called bivariate normal distribution may be used as a model for any bivariate distribution. A bivariate distribution, however, is not determined by its two margins. Even if they are normal, the bivariate density function need not be the usual exponential function of a quadratic form in the two variables X and Y. The regression curves need not be linear and the curves of equal probability density need not be ellipses. On the contrary, there corresponds to given normal margins an infinity of bivariate distributions. A general form has been developed for effecting the constructi of a bivariate distribution for any given margins (publicati 18).

A family of bivariate probability functions with specified marginal probability functions $F_1(X) = 1 - P_1(X)$ and $F_2(Y) = 1 - P_2(Y)$, with parameter a, $-1 < a < 1$, is given by

$$F(X,Y) = F_1(X)\, F_2(Y) \left[1 + aP_1(X)\, P_2(Y)\right] . \tag{3}$$

The parameter value $a = 0$ corresponds to independence. In the normal case the regression curves are linear functions of the probabilities and the correlation coefficient ρ is bounded by $-1/\pi < \rho < 1/\pi$. Tables of the bivariate probability function for given numerical values of the margin have been computed, where ρ is limited to the domain $-1/3 < \rho < 1/3$. Generalizations to higher dimensions follow readily. Technical Report 12 and publications 32 and 35 deal with these topics.

Methods have been developed for constructing bivariate distributions from exponential and logistic margins. The resulting distributions are quite different from those previously obtained by statisticians. Technical Reports 15 and 18 and publications 42 and 46 present this material.

II. STATISTICAL THEORY OF EXTREME VALUES

In 1958 the Columbia University Press published E. J. Gumbel's book, Statistics of Extremes, which provides a systematic and comprehensive presentation of the statistical theory of extremes up to that date. The book was well received by both the profession and reviewers, and a second printing was made in 1959.

The asymptotic distribution of the range was generalis to the m^{th} range in Report 11 and publication 30; the m^{th} range is the difference between the m^{th} value from the top and the m^{th} value from the bottom. In a large sample the asymptotic density function of the m^{th} range can be reduced to the corresponding function of the range proper, i.e. to a Bessel function. The m^{th} mid-range is the sum of the m^{th} extremes. It is shown (in Report17) that its distribution is a generalisation of the logistic distributi The distributions of the m^{th} range and the m^{th} mid-range converge to normality with increasing rank m.

While it is known that the quantiles are asymptoticall normally distributed and the two extremes are independent, the joint asymptotic distribution of a quantile and of the extremes was not known. This distribution was shown by Professor Berman (Report 22) to split, with increasing sample size, into the product of the normal distribution of the quantile and the extremal distribution, demonstrati the asymptotic independence of these two statistics.

Some results on the rejection of outliers by means of extreme value theory have been presented in publication Number 36. Extreme value theory seems to be the natural approach to the detection of outliers.

Considerable effort was devoted to study of the asymptotic bivariate and multivariate extremal distributior (Report 19 and publication 41). In the simplest case the bivariate distributions are the products of the correspondi univariate functions (publication 14).

In the bivariate case, there are three special cases ari the three extremal distributions to consider.

There are, therefore six-fold infinities of such distributions(publication 14) General formulae which contain undetermined functions of th two marginal extremal probabilities are interesting from a theoretical point of view but are not of much practical use What is needed are specific cases where the bivariate extremal distributions can be written down expressed by the univariate distributions and a factor depending on the correlation of the two marginal distributions. Two such cases are given in publications 39 and 44. Let

$$(4)\quad -\log \Phi^{(\kappa,\lambda)}_{(x,y)} = \zeta\ ;\ -\log \Phi^{(\kappa)}_{(x)} = \xi\ ;\ -\log \Phi\left\{{\lambda \atop y}\right\} = \eta$$

be transformations of the bivariate and the two marginal extremal distributions, then the two cases are given by

$$(5)\quad \zeta^{m} = \xi^{m} + \eta^{m} \qquad ;\ m > \text{or} = 1$$

and

$$(6)\quad \xi = \eta + \zeta + \alpha\left(1/\xi + 1/\eta \right)^{-1} \qquad 0 \leq \alpha \leq 1$$

The same formulae hold for bivariate extremal distributions of smallest values. The special cases m = 1 and a = 0 stand for independence. The formulae can easily be generalized into more than two dimensions.

III. Applications to Breaking Strength and Fatigue of Materials

The study of fatigue and breaking strength of metals is an important field for the application of extreme value theory (publication 4). A popular resumé of the numerous aspects and technical applications in this domain was given in Report 7. There is given, in particular, an analysis of the corrosion problem, i.e. the pit depth of pipe lines, which is of great importance for the transportation of oil: the maximum pit depth increases as a linear function of the length of the line.

Previously only classical methods of statistics and curve fitting were applied to such problems. An important development in the analysis of fatigue was made by Weibull who used the third asymptotic distribution of smallest values, although on a purely empirical basis. The statistical analysis takes the number of cycles at failure under a given stress as a random variable. Although the stresses at failure are constant within each experiment it is proper to interpret them also as random variables and to use the same theory for both problems. These notions have been developed in Reports 8 and 9 and publications 8, 9, 10, and 13, where more than thirty sets of data on fatigue failure were analyzed. The most important characteristics are as follows: a) The minimum life $N_{o,s}$, i.e. the largest number of cycles with probability one of survival, which decreases with increasing stress (publication 26). Its inverse $S_{o,N}$ is the stress with probability of survival one. b) The endurance limit S_o, i.e. the stress so small that the specimen may survive an infinity of cycles, c) the probability of permanent survival $\mathcal{L}(S,\infty)$ is a functic of the stress. None of these values can be observed. In the present theory they take on the role of parameters whick can be estimated if a sufficiently large sample of observati is made. The statistical theory is based on the two conditional survivorship functions

$$(7) \qquad \mathcal{L}(N|S) = \exp\left[-\left(\frac{N - N_{o,s}}{V_s - N_{o,s}}\right)^{\alpha_s}\right]$$

$$(8) \qquad \mathcal{L}(S|N) = \exp\left[-\left(\frac{S - S_{o,N}}{S_V - S_{o,N}}\right)^{\beta_N}\right]$$

where the parameters $N_{o,s}$ and S_o are as explained previously

The number of cycles V_s and its inverse, the stress S_V are parameters of location while α_s and β_N are parameters of scale and shape. Formula 8 can also be used for the probability of permanent survival. The analysis of these parameters and their interdependence is contained in Report 20 and publication 21. The parameters of scale which have no dimensions are linked to the lower limits.

This statistical approach was contrasted to the purely empirical procedures used by Bastenaire and Bennett in publication 12. Several semi-popular articles (publications 5 and 15) have been written on request.

While experiments may be performed so that we approach the minimum life no experiment is possible to approach the endurance limit. Instead of an infinity of cycles, a large number, 10^7 or possibly 10^8, is used for its estimation. The existing estimates of the endurance limits are based on an apparent discontinuity of the survivorship functions while the statistical theory must assume continuity. Therefore the estimates given in the literature may be of doubtful value. The problem of the endurance limit is treated in publication 6 and a test program appropriately designed for the estimation of the endurance limit is given in publication 2. Unfortunately no such large scale testing program has ever been undertaken.

A resumé of the logical, physical and statistical reasons for preferring the extreme value theory in this analysis is given in publication 47.

IV. Applications to Hydrology

The first large scale application of the theory of extreme values arose from hydrological problems, in particular the analysis of floods and droughts. By their very definition they are the annual largest and smallest values of the daily discharges of a river at a given station, hence extreme values.

At the invitation of the Société Hydrotéchnique de France, E. J. Gumbel wrote expository papers explaining his method of analysis, in particular the use of extreme value paper in graphical procedures for the analysis and forecast from drought and flood data (publications 7, 19, 31). Using the return period scale for forecast, an analysis of records of twenty-five rivers provided fruitful examples of the use of these methods. In the course of the extensive discussion that arose from these papers, the question was raised as to why the first asymptotic distribution seemed to represent adequately the distribution of largest annual discharges (with occasional better representation by the second distribution). A further question was raised, namely, why in the case of minimum discharges there could be, for some rivers, positive asymptotes and for other rivers zero least value.

At the invitation of the British Journal of the Institution of Water Engineers, an article was written in explanation of both these questions. In response, methods for constructing linear control boundaries which are useful in extrapolation and a graphical method for selecting among the three types of extremal distribution that type most effective for representing a given sequence of data were presented (Report 13, publication 16). The method of discrimination is based on the curvature which is assumed to hold for the whole sequence. In consequence of this analysis, it is shown that the first or second distributions of largest values is adequate for floods. So far, no example has been found where the third distribution (which has an upper limit) is indicated. This result is interesting in view of the fact that the objection of many engineers to the statistical theory of extreme values is made precisely because they feel that the appropriate distribution should have a finite upper bound On the other hand, as might be expected, the third asymptot: distribution of the smallest values is useful in representi:

droughts. Further interest in the question of the most appropriate statistical representation of floods and droughts by way of communications from various countries was responded to in Report 1 and publication 27.

An important doubt as to the validity of using the present statistical theory of extremes which is based on an assumption of the independence of successive observations of daily discharges has been largely mitigated in work of Professor Berman who showed that the actual absence of independence in the observations did not invalidate the results obtained by the present methods. For further information, a list of recent papers on Extreme Values is added as Appendix E.

APPENDIX A

Personnel Who have served on the Scientific Staff of this C

Professor E. J. Gumbel, Principal Investigator
Professor S. B. Littauer, Project Director

Simeon Berman, now Assistant Professor, Department of Mathem
Statistics, Columbia University
Cyrus Derman, now Associate Professor, Department of Industr
Engineering, Columbia University
Neil Goldstein, graduate student in mathematical statistics

T. T. Kuo, obtained Ph.D. in Industrial Engineering, Columbi
University, 1961, now in Operations Research,
National Cash Register Co., Dayton, Ohio
Seiti Sugihara, then a graduate student in mathematical stat
Taro Yamane, since Assistant Professor of Statistics, New
York University
Phillip G. Carlson, received D.Eng.Sci. in Industrial Engine
1962, now on Operations Research staff, Arthu
Andersen and Co., New York; co-operated with
Professor E. J. Gumbel on researches under
that contract without formal appointment
to the staff.
José Tiago de Oliveira, Professor of the Faculty of Sciences
Lisbon, Portugal.

APPENDIX B - TECHNICAL REPORTS

1. E. J. Gumbel - MINIMUM LIFE IN FATIGUE FAILURES

2. C. Derman, T. T. Kwo and E. J. Gumbel
STANDARD ERRORS OF ESTIMATE OF PARAMETERS OF FATIGUE FAILURE SURVIVORSHIP FUNCTIONS

3. E. J. Gumbel - STATISTICAL ESTIMATION OF THE ENDURANCE LIMI

4. C. Derman - SOME REMARKS ON THE ENDURANCE LIMIT PROBLEM

5. S. Sugihara - SOME TESTS FOR MINIMUM LIFE OF FATIGUE FAILU SURVIVORSHIP FUNCTIONS

6. E. J. Gumbel - STATISTICAL ESTIMATION OF THE ENDURANCE LIMI

7. E. J. Gumbel - EXTREME VALUES IN TECHNICAL PROBLEMS (Reprin from Industrial laboratories, Vol. 7, no. 12 Dec., 1956)

8. E. J. Gumbel - STATISTICAL ANALYSIS OF FATIGUE

9. E. J. Gumbel - THE STATISTICAL ASPECT OF FATIGUE (Reprint f Columbia Engineering Quarterly, March 1957, pp. 3-7)

10. E. J. Gumbel + P. G. Carlson
ON THE ASYMPTOTIC COVARIANCE OF THE SAMPLE M AND STANDARD DEVIATION (Reprint from Metron, vol. 13, No. 1, pp. 1-9, Roma, 1956)

11. E. J. Gumbel - THE M^{TH} RANGE

12. E. J. Gumbel - MULTIVARIATE DISTRIBUTIONS WITH GIVEN MARGIN

13. E. J. Gumbel - STATISTICAL THEORY OF FLOODS AND DROUGHTS (Reprint from Journal of the Institution of Water Engineers, Vol. 12, pp. 157-184, Londo

14. E. J. Gumbel - MEASUREMENTS OF RARE EVENTS

15. E. J. Gumbel - BIVARIATE EXPONENTIAL DISTRIBUTIONS

16. E. J. Gumbel, A. Avishur, A. D. Benham, F. Law, R. W. S. Thompson, and D. H. Thompson
COMMUNICATIONS ON THE STATISTICAL THEORY OF FLOODS AND DROUGHTS (Reprint from Journal of the Institution of Water Engineers, Vol. 13,

17. E. J. Gumbel - STATISTICAL THEORY OF EXTREME VALUES (MAIN RESULTS)

18. E. J. Gumbel - A BIVARIATE LOGISTIC DISTRIBUTION

19. E. J. Gumbel - MULTIVARIATE ASYMPTOTIC DISTRIBUTIONS OF EXTREME VALUES

20. E. J. Gumbel - ON THE PARAMETERS OF THE DISTRIBUTION OF FATIGUE LIFE

21. E. J. Gumbel - THE RETURN PERIOD OF ORDER STATISTICS

22. S. Berman - THE ASYMPTOTIC INDEPENDENCE OF THE SAMPLE QUANTILE AND THE SAMPLE EXTREME VALUE

APPENDIX C - PUBLICATIONS

1. Review of Epstein, Truncated life tests (Ann. Math. Stat. Vol. 25, 1954.) Math. Tables and Aids to Calc., Vol. 9, Washingt

2. Programm fur die statistische Schatzung der Dauerschwingfestig Schweizer Archiv fur Wiss, u. Technik, Vol. 22, No. 11, p. 3 November, 1956.

3. Methodes graphiques pour l'analyse des debits de crue. La Hou Blanche, Numero 5, pp. 709-717, Paris, November 1956.

4. Statistician attacks extreme values in technical problems. In trial Laboratories, Vol. 7, No. 12, Chicago, December 1956.

5. Statistische Deutung der Ermidungserscheinungen bei Metallen. Schweizer Technische Rundschau, Vol. 49, No. 3, pp. 3-7, Ber January 1957.

6. Statistical estimation of the endurance limit. Proceedings of Second Annual Statistical Engineering Symposium, Chemical Co Engineering Command, Army Chemical Center, pp. 63-86, Maryla 26-27 April 1956.

7. Methodes graphiques pour l'analyse des debits de crue. Revue Statistique Appliquee, Vol. 5, No. 2, pp. 77-89, Paris, 1957

8. The statistical aspect in fatigue in Proceedings 2nd Conferenc the mechanics of elasticity and plasticity. Sponsored by Of of Ordnance Research, pp. 356-366, Washington, Febmuary 1957

9. Discussion on fatigue. Second Conference on the Mechanics of Elasticity and Plasticity. Sponsored by Office of Ordnance U. S. Army, Durham, North Carolina, pp. 25, 52, Washington, February 1957.

10. The statistical aspect of fatigue. Columbia Engineering Quart pp. 22-25, p. 60, March 1957.

11. Statistical distribution patterns of ocean waves. Trans. Soci of Naval Architects and Marine Engineers, 8, p.427, 1956.

12. Discussion of the contributions of Mr. Bastenaire and Mr. Benn International Conference on Fatigue of Metals, British Insti of Mechanical Engineers, London 1956.

13. Etude statistique de la fatigue des materiaux. Revue de Stati Appliques, Vol. 5, No. 4, pp. 51-86, Paris 1957.

14. Fonctions de probabilites a deux variables extremales independ C.R.Ac.Sc., Vol. 246, pp. 49-50, Paris 1958.

15. Interpretation statistique de la fatigue, Colloques de Calcul numerique, Dijon 1956, Publications Scientifiques et Techniques du Ministere de l'Air, No. N.T. 77, pp. 9-21, Paris 195

16. Statistical theory of Floods and Droughts, Journal of the Institution of Water Engineers, Vol. 12, No. 3, pp. 157-184, May 19

17. Statistician Abroad. Ordnance Research Bulletin, Durham, N. C., April-May 1958.

18. Distributions a plusieurs variables dont les marges sont donnees With remarks by M. Frechet. C. R. Ac. Sc. Vol. 246, pp. 2717-2720, Paris 1958.

19. Discussion of "The Statistical Treatment of Flood Flows," Trans. Amer. Geophysical Union, Vol. 39, No. 4, pp. 732-733, Washington 1958.

20. Review of "Dictionary of Statistical Terms," by M. G. Kendall an W. E. Buckland, Biometrika, Vol. 45, p. 283, London 1958.

21. Various Aspects of the Distribution of Fatigue Lives, Wright Air Development Center, WADC Technical Report 58-72, ASTIA Documen No. 155747, 40 pages, July 1958.

22. Statistics of Extremes, Columbia University Press, 375 pages, New York 1958.

23. List of Publications of E. J. Gumbel, Statistica, 18, No. 3, pp. 563-571 Rome, Italy 1958.

24. Contributions to the discussions of the International Statistica Institute, Stockholm Congress, 1957, Bulletin de l'Institut International de Statistique, Vol. 36, Part 1, pp. 66, 70, 87, 111, Stockholm, 1959.

25. Congress of the International Statistical Institute, Ordnance Research Bulletin, ORD-59-5, 59-6, May, June 1959, Durham, North Carolina.

26. Le Phenomene de la vie minima en fatigue sous des efforts constantes et variables, with A. M. Freudenthal, Revue de metallurgie, Vol. 56, no. 3, pp. 295-298, Paris, 1959.

27. Communications on the Statistical Theory of Floods and Droughts, Journal of the Institution of Water Engineers, Vol. 13, No. 1, pp. 86-112, London, February, 1959.

28. Statistical Theory of Extreme Values, Bulletin, International Statistical Institute, Vol. 36, Part 3, pp. 12-14, Stockholm,

29. Measurement of Rare Events, Chapter 11, of Measurement : Definitions and Theories, edited by C. W. Churchman and P. Ratoosh, pp. 204-217, John Wiley and Sons, Inc., New York 1959.

30. The m^th range, Journal de Mathematiques, Vol. 39, No. 3, pp. 253-265, Paris, 1959.

31. Theorie statistique des debits d'etiage, La Houille Blanche, Vol. 14, pp. 57-65, Grenoble, January-February, 1959.

32. Multivariate Distributions with Given Margins, Revista da Faculdade de Ciencias de Lisboa, 2. Serie-A-vol. VII-Fasc. 2, pp. 179-218, Lisboa, 1959.

33. Statistics of Extremes, 2nd printing, Columbia University Press, 1960, New York.

34. Reports on European Science, Ordnance Research Bulletin, ORB-60-60-4, February, April, 1960, Durham, North Carolina.

35. Multivariate distributions with given margins and analytical examples. 31st Session of the International Statistical Institute, Bruxelles, 1960, pp. 363-373, Bulletin de l'Instit International de Statistique, Bruxelles, 1960.

36. Discussion, F. J . Anscombe's paper, Rejection of Outliers, Tec nometrics, Vol. 2, No. 2, May 1960, pp. 165-166, Washington,

37. Review of Gunnar Blom's book, Statistical Estimates and Transfo Beta-Variables, in: Canadian Mathematical Bulletin, Vol. 3, no. 1, p. 201, Montreal 1960.

38. Review of C. Berge's book, Theorie des Graphes et ses Applications, in: Management Science, Vol. 7, No. 1, p. 88, Baltimo

39. Multivariate extremal distributions, Abstract, Ann. Math. Stat. Vol. 31, p. 1216, 1960.

40. Discussion to the 31st Session of the International Statistical Institute, Bulletin de l'Institut International de Statistiqu Vol. 37, No. 1, pp. 6, 79, 99, 114, Bruxelles, 1960.

41. Distributions des valeurs extremes en plusieurs dimensions, Publications de l'Institut de Statistique de l'Universite de Paris, Vol. 9, No. 2, pp. 171-173, 1960.

42. Bivariate exponential distributions, Journ. Am. Stat. Assoc., Vol. 55, pp. 698-707, 1960.

43. The return period of order statistics, Ann. Inst. Stat. Mat., Vol. 12, No. 3, pp. 249-256, Tokyo, 1960.

44. Multivariate extremal distributions, Bulletin de l'Institut International de Statistique, Preprint, Paris 1961.

45. Report on a Scientific trip to East Asia, Ordnance Research Bulletin, ORB-61-6, 1961

46. Bivariate logistic distributions, Journ. Am. Stat. Assoc., Vol. 56, pp. 335-349, 1961.

47. Statistical theory of breaking strength and fatigue failure, Bulletin de l'Institut International de Statistique, Vol. 38, No. 3, pp. 275-393, Tokyo, 1961.

APPENDIX D

Other Scientific Work Done on This Contract

Dr. E. J. Gumbel made a number of trips abroad, disseminating the ideas developed in the researches sponsored by this contract. Descriptions of these activities have been published in articles 17, 25, 34 and 45. He also participated in a number of international scientific congresses, reported on in articles 24 and 40.

A by-product of these researches is a course on the Statistical Theory of Extreme Values and their Technical Applications, given at Columbia University (Catalogue Number: E 6611 x - E 6612 y. Engineering Applications of Extreme Values).

Professor S. B. Littauer (jointly with Professor S. Ehrenfeld) has worked on a text on Statistical Method in Engineering and Science, which is scheduled for publication early in 1963.

APPENDIX E

Publications Concerning Gumbel's Work 1956-1961

1. Jacques Bernier: Sur l'application des diverses lois limites des valeures exrmes au probleme des debits de crue. La Houille Blanche, No. 5, pp. 718-725, Grenoble, November 1956

2. Howard J. Pincus: Some vector and arithmetic operations on two dimensional orientation variates, with applications to geological data. The Journal of Geology, Vol. 64, No. 6, pp. 533-557, November 1956

3. G. S. Watson and E. J. Williams: On the construction of signif cance tests on the circle and the sphere. Biometrika, Vol. 4 Parts 3 and 4, pp. 344-352, December 1956

4. K. H. Weber and H. S. Endicott: Area effect and its extremal basis for the electric breakdown of transformer oil. Power Apparatus and Systems, pp. 371-381, June 1956

5. K. H. Weber and H. S. Endicott: Electrode area effect for the impulse breakdown of transformer oil. AIEE Transactions, Vol. 76, Part III, 1957

6. K. H. Weber and H. S. Endicott: Extremal area effect for large area electrodes for the electric breakdown of transformer oil AIEE Transactions, Vol 76, Part III, 1957

7. J. Tiago de Oliveira: Estimators and tests for continuous popu lations with locations and dispersion parameters. Revista da Faculdade de Ciencias de Lisboa, 2,a Serie-A-Vol. VI, Fasc. 1 pp. 121-146, Lisbon, 1957

8. K. L. Brakensiek and A. W. Zingg: Application of the extreme v statistical distribution to annual precipitation and crop yie ARA 41-13, U. S. Dept of Agriculture, Washington, February 19

9. W. D. Potter: The effect of nonrepresentative sampling on line regressions as applied to runoff. Trans. Am. Geophys. Union, Vol. 38, No. 3, pp. 333-340, June 1957

10. Ralph R. Botts: "Extreme-value" methods simplified. Agricultu Economics Research, U. S. Dept of Agriculture, Vol. IX, No. 3 pp. 88-95, July 1957

11. Bruno Carene: Longevité et fontaines de jouvence. Diagrammes. 6., pp. 16-20, Editions du cap., Monte-Carlo, August 1957

12. D. Van Dantzig: Mathematical Problems Raised by the Flood Disaster 1953. Proceedings of the International Congress of Mathematicians, 1954, Vol. I, pp. 218-239, Amsterdam, 1954

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